

GEOMETRY A  
FINAL EXAM REVIEW

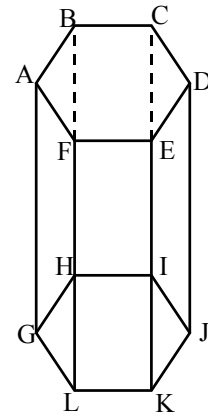
NAME           **KEY**          

UNIT I: INTRODUCTION TO GEOMETRY

1. Name the three undefined terms of geometry.  
***Point, line, and plane***
- 

2. Given the diagram of a right hexagonal prism, determine whether each statement is true or false.

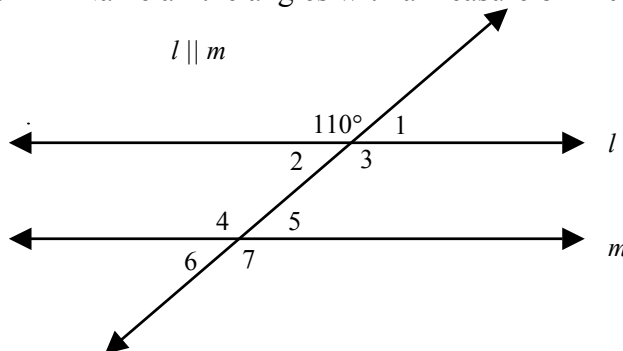
- a. A, B, and C are collinear. ***False***  
 b. D, E, K, and J are coplanar. ***True***  
 c. B and J are collinear. ***True***  
 d. E, F, J, and K are coplanar. ***False***



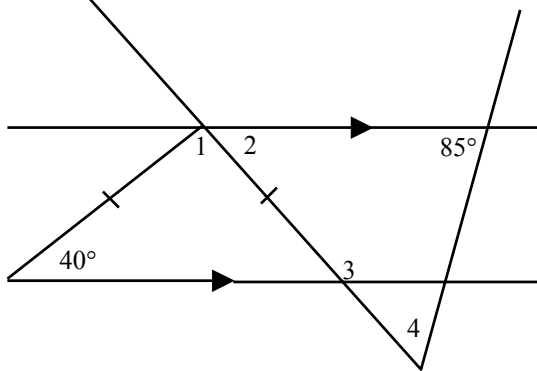
3. Xena lives 15 blocks from Yolanda and Yolanda lives 5 blocks from Zuri. Given all three houses are collinear, which one of the following locations of points is **NOT** possible?

- A.  $\overline{X \quad Y \quad Z}$                       B.  $\overline{X \quad Z \quad Y}$   
 C.  $\overline{Y \quad X \quad Z}$
- 

4. Name all the angles with a measure of  $110^\circ$ .                       **$\angle 3, \angle 4, \angle 7$**



5. Find the measures of the numbered angles. Use mathematics to explain the process you used to determine the measures. Use words, symbols, or both in your explanation.



$m\angle 1 = \underline{100^\circ}$

$m\angle 2 = \underline{40^\circ}$

$m\angle 3 = \underline{140^\circ}$

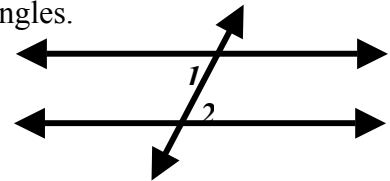
$m\angle 4 = \underline{55^\circ}$

6. Complete the following statements.  
 a. The ceiling and the floor of our classroom are examples of parallel planes.  
 b. The wall and the floor of our classroom are examples of perpendicular planes.

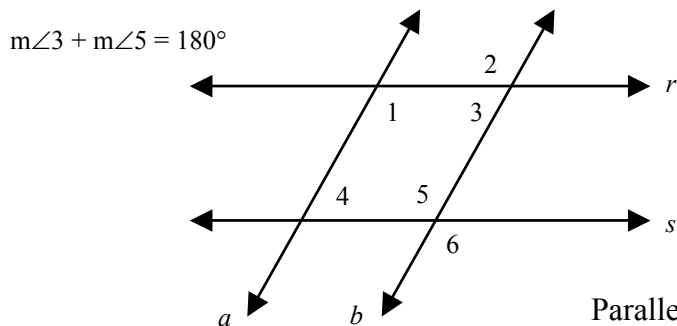
7. Two lines that do not lie in the same plane are called skew lines.

8. Make a sketch that illustrates a pair of alternate interior angles.

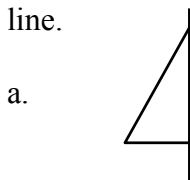
*$\angle 1$  and  $\angle 2$  are alternate interior angles*



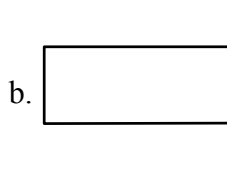
9. Use the figure below and the given information to determine which lines are parallel. Use mathematics to explain the process you used to determine your answer. Use words, symbols, or both in your explanation.



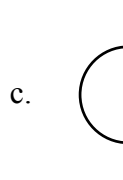
10. Name the solid of revolution formed when the given figure is rotated about the line.



Cone



Cylinder

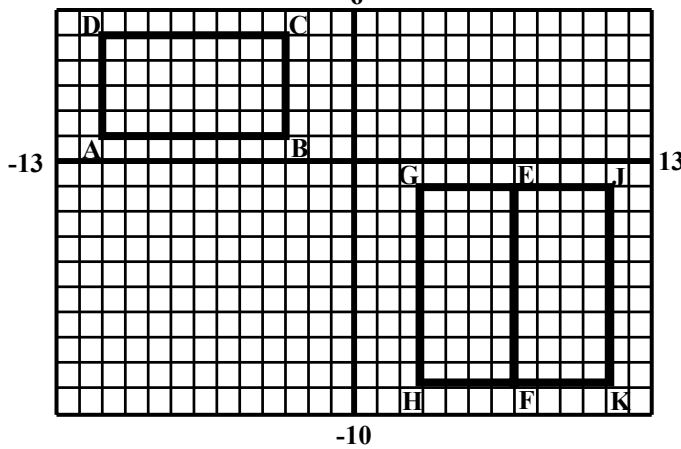


Sphere



Torus or Donut shape

11. If  $\overline{EF}$  is congruent to  $\overline{AB}$ , then how many rectangles with  $\overline{EF}$  as a side can be drawn congruent to rectangle ABCD? 2



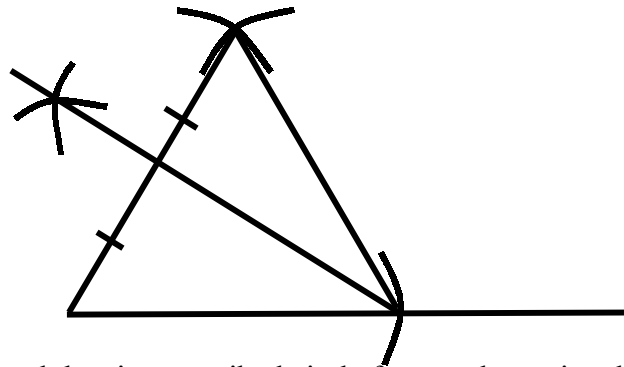
Provide a sketch. Label and give the coordinates for the vertices of each rectangle.

- E (7, -1)
- F (7, -9)
- G (3, -1)
- H (3, -9)
- J (11, -1)
- K (11, -9)

12. If a plane were to intersect a cone, which of the following could **NOT** represent the intersection?  
 A. Circle      **(B)** Rectangle      C. Ellipse      D. Line      E. Point

13. If a plane were to intersect a cylinder, which of the following could **NOT** represent the intersection?  
 A. Circle      B. Rectangle      **(C)** Trapezoid      D. Line      E. Point

14. Construct an equilateral triangle with a median. Use mathematics to explain the process you used for your construction. Use words, symbols, or both in your explanation.

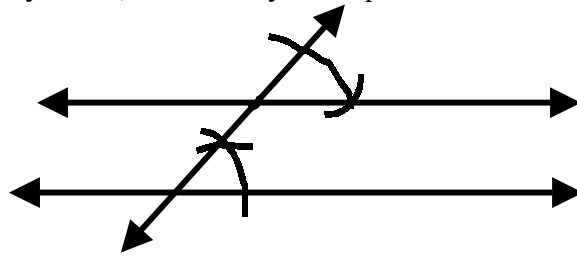


- 15H. Construct the inscribed circle and the circumscribed circle for a scalene triangle. Use mathematics to explain the process you used for your construction. Use words, symbols, or both in your explanation.

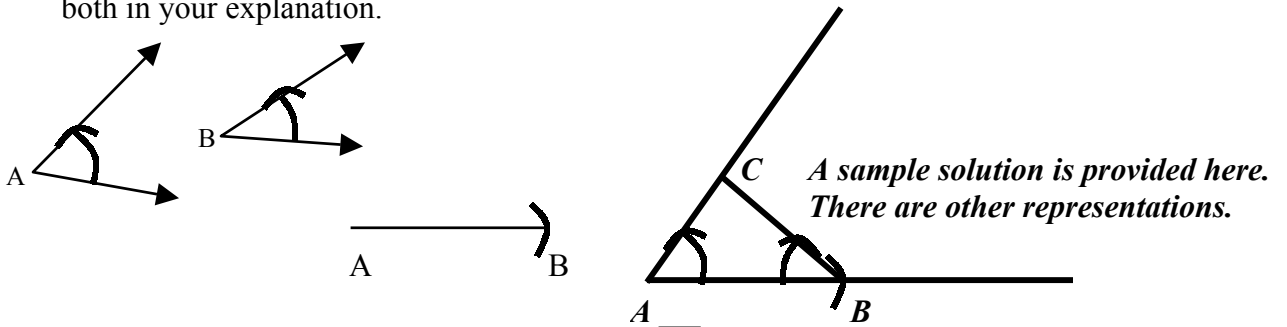
**Student needs to construct perpendicular bisectors of the sides of the triangle to find the center of the circumscribed circle (this center is equidistant from the vertices of the triangle) and needs to construct angle bisectors of the triangle to find the center of the inscribed circle (this center is equidistant from the sides of the triangle.) Students should then draw the appropriate circle.**

16. Construct a pair of parallel lines. Use mathematics to explain the process you used for your construction. Use words, symbols, or both in your explanation.  
BCR

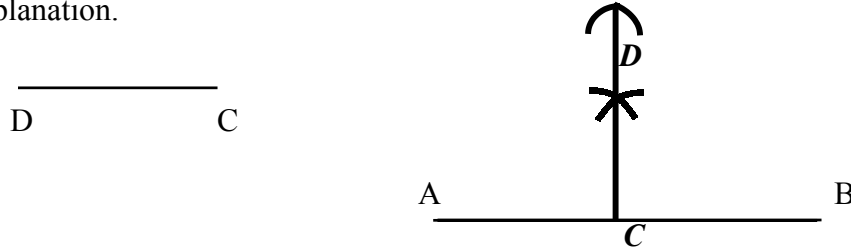
*A sample solution is provided here.  
There are other representations.*



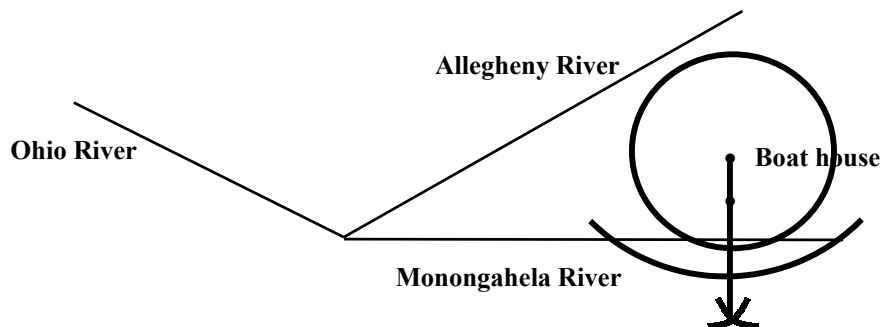
17. Using the angles and segment below, construct triangle ABC. Use mathematics to explain the process you used for your construction. Use words, symbols, or both in your explanation.  
BCR



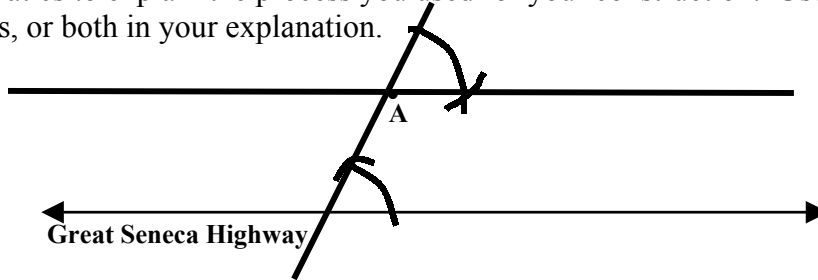
18. Construct  $\overline{DC}$  as the perpendicular bisector of  $\overline{AB}$ . Use mathematics to explain the process you used for your construction. Use words, symbols, or both in your explanation.  
BCR



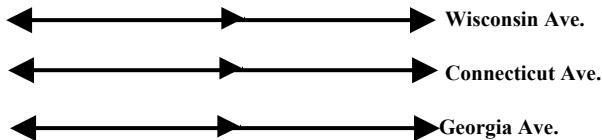
19. The crew team wants to walk from their boat house to the nearest river. Show by construction which river is closest to the boat house. Construct the shortest path to that river. Use mathematics to explain the process you used to determine your answer. Use words, symbols, or both in your explanation.  
ECR



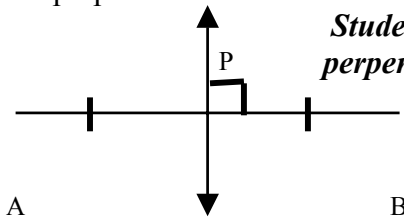
20. BCR A civil engineer wishes to build a road passing through point A and parallel to Great Seneca Highway. Construct a road that could meet these conditions. Use mathematics to explain the process you used for your construction. Use words, symbols, or both in your explanation.



21. If Wisconsin Avenue is parallel to Connecticut Avenue and Connecticut Avenue is parallel to Georgia Avenue, then what relationship exists between Wisconsin Avenue and Georgia Avenue? ***They are parallel.*** (Provide a sketch.)

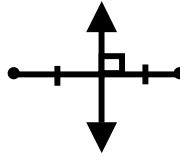


22. ECR Using a flow chart, paragraph, or two-column proof, prove why any point P on the perpendicular bisector of  $\overline{AB}$  is equidistant from both points A and B.

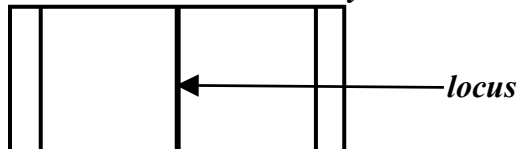


*Student's proof should indicate choosing a point on the perpendicular bisector, not on segment AB, and proving congruent triangles.*

23. Sketch and describe the locus of points in a plane equidistant from two fixed points. ***The locus is the perpendicular bisector of the segment that connects those two points.***



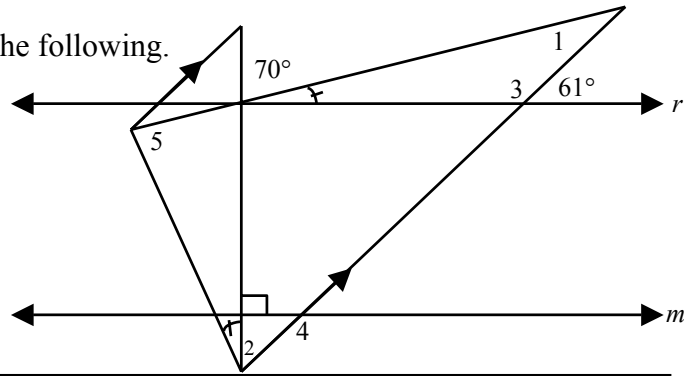
24. Sketch and describe the locus of points on a football field that are equidistant from the two goal lines. ***The locus is the 50 yard line on the football field.***



26. ECR Can you construct a  $45^\circ$  angle with only a compass and straight edge. Use mathematics to explain the process you could use to construct the angle. Use words, symbols, or both in your explanation. ***Yes. Construct two perpendicular lines. Then construct an angle bisector of one of the right angles formed by the two perpendicular lines.***

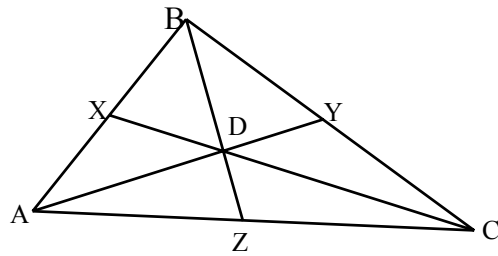
27. If  $r \parallel m$ , find the measure of the following.

$$\begin{aligned} m\angle 1 &= \underline{41^\circ} \\ m\angle 2 &= \underline{29^\circ} \\ m\angle 3 &= \underline{119^\circ} \\ m\angle 4 &= \underline{119^\circ} \\ m\angle 5 &= \underline{90^\circ} \end{aligned}$$

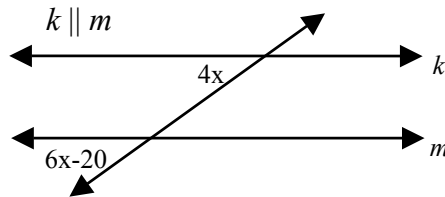


28H. D is the centroid in the figure to the right.  
 $BD = 10$ ,  $DY = 4$ , and  $CD = 16$ .  
 Find the following.

$$\begin{aligned} DX &= \underline{8} \\ AY &= \underline{12} \\ BZ &= \underline{15} \end{aligned}$$



29. Find the value of  $x$  in the diagram below.

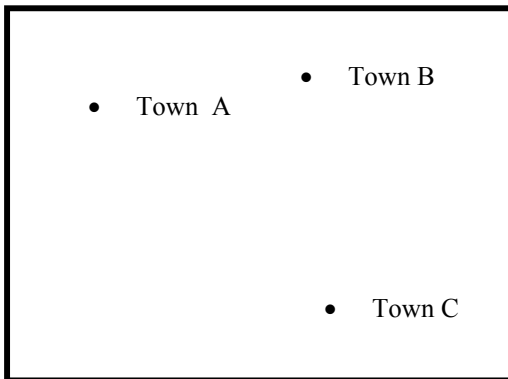


*Since  $4x$  and  $6x - 20$  are corresponding angles their angle measures are equal.*

*Therefore,*

$$\begin{aligned} 4x &= 6x - 20 \\ 20 &= 2x \\ x &= 10 \end{aligned}$$

30. ECR The Department of Public Works wants to put a water treatment plant at a point that is an equal distance from each of three towns it will service. The location of each of the towns is shown below. **Complete the following (you may need separate paper).**



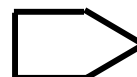
- Locate the point that is equal in distance from each of the towns. *See description below*
- Explain how you determined this location. Use words, symbols, or both in your explanation.  
*I connected the towns (vertices) to make a triangle. I constructed the perpendicular bisectors of each side of the triangle. The three perpendicular bisectors intersect at a point called the circumcenter. The circumcenter is the location of the water treatment plant.*
- Use mathematics to justify your answer.  
*Since the circumcenter is equidistant from all the vertices of a triangle, I knew to construct that point to solve the problem. I constructed the perpendicular bisectors because the circumcenter of a triangle is the point of concurrency that is formed by the intersection three perpendicular bisectors.*

UNIT II: EXPLORING GEOMETRIC RELATIONS AND PROPERTIES

31. Place check marks in the boxes where the property holds true.

| Property                                     | Parallelogram | Rectangle | Square | Rhombus | Trapezoid |
|--|---------------|-----------|--------|---------|-----------|
| 1. Opposite sides congruent                  | ✓             | ✓         | ✓      | ✓       |           |
| 2. Opposite sides parallel                   | ✓             | ✓         | ✓      | ✓       |           |
| 3. Opposite angles congruent                 | ✓             | ✓         | ✓      | ✓       |           |
| 4. Each diagonal forms 2 congruent triangles | ✓             | ✓         | ✓      | ✓       |           |
| 5. Diagonals bisect each other               | ✓             | ✓         | ✓      | ✓       |           |
| 6. Diagonals congruent                       |               | ✓         | ✓      |         |           |
| 7. Diagonals perpendicular                   |               |           | ✓      | ✓       |           |
| 8. A diagonal bisects two angles             |               |           | ✓      | ✓       |           |
| 9. All angles are right angles               |               | ✓         | ✓      |         |           |
| 10. All sides are congruent                  |               |           | ✓      | ✓       |           |

32. Sketch a pentagon that is equilateral but not equiangular.



33. ECR A regular polygon has exterior angles that measure  $60^\circ$  each. Determine the sum of the measures of the interior angles of the polygon in degrees and the measure of one interior angle. Use mathematics to explain the process you used to determine your answer. Use words, symbols, or both in your explanation.

***6 sided figure  $720^\circ$   $120^\circ$***

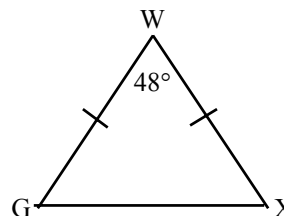
34. Find the measure of  $\angle G$ .

$m\angle G = \underline{66^\circ}$

$$48^\circ + b + b = 180^\circ$$

$$2b = 132^\circ$$

$$b = 66^\circ$$



35. Find the measure of  $\angle PMQ$ . Use mathematics to explain the process you used to determine your answer. Use words, symbols, or both in your explanation.

$8x - 43 = 2x + 25 + 2x$   
 $8x - 43 = 4x + 25$   
 $4x = 68$   
 $x = 17$   
 $8(17) - 43 = 93^\circ$

$m\angle PMQ = \underline{93^\circ}$

36. Four interior angles of a pentagon have measures of  $80^\circ$ ,  $97^\circ$ ,  $104^\circ$ , and  $110^\circ$ . Find the measure of the fifth interior angle. 149
- $(5 - 2)180 = 540$   
 $540 - (80 + 97 + 104 + 110) = 149$

37. If each interior angle of a regular polygon has a measure of  $160^\circ$ , how many sides does the polygon have?

$$\frac{(n-2)180}{n} = 160$$

$$n = 18$$

*The polygon has 18 sides.*

38. The bases of trapezoid ABCD measure 13 and 27, and  $\overline{EF}$  is a midsegment:

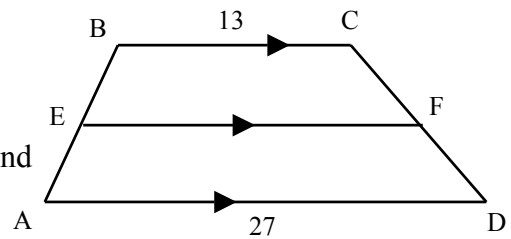
- a. What is the measure of  $\overline{EF}$ ?

**EF = 20**

- b. Find AB if trapezoid ABCD is isosceles and  $AB = 5x - 3$  when  $CD = 3x + 3$ .

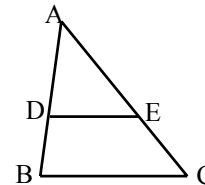
$5x - 3 = 3x + 3$   
 $x = 3$

$AB = \underline{12}$        $AB = 5(3) - 3 = 12$       *Check:  $CD = 3(3) + 3 = 12$*



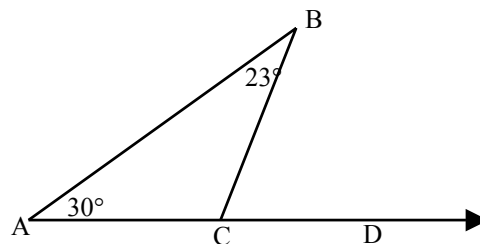
39. Given  $\triangle ABC$  with midsegment  $\overline{DE}$ . If  $BC = 28$ ,  $DE =$

**14**



40. Find the measure of  $\angle BCD$ .

$23^\circ + 30^\circ = 53^\circ$   
 $m\angle BCD = \underline{53^\circ}$

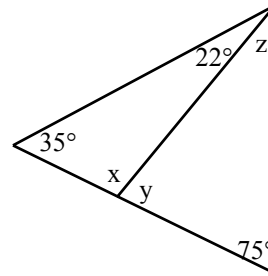




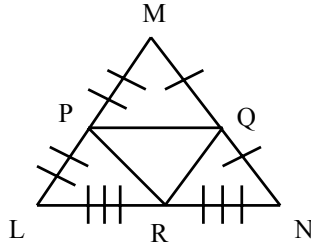
41. Find the measures of angles  $x$ ,  $y$ , and  $z$ .

$$\begin{aligned} x &= \underline{123^\circ} & X &= 180^\circ - (22^\circ + 35^\circ) = 123^\circ \\ y &= \underline{57^\circ} & Y &= 180^\circ - 123^\circ = 57^\circ \\ z &= \underline{48^\circ} & Z &= 180^\circ - (57^\circ + 75^\circ) = 48^\circ \end{aligned}$$

*(other methods possible)*



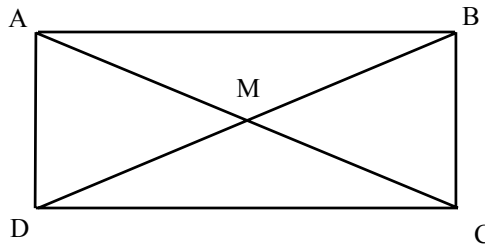
42. For the triangle shown, if  $QR = 8.5$ , what is the measure of  $\overline{ML}$ ? 17



$$\begin{aligned} ML &= 2(QR) \\ ML &= 2(8.5) \\ ML &= 17 \end{aligned}$$

43. ABCD is a rectangle with diagonals  $\overline{AC}$  and  $\overline{BD}$  that intersect at point M.  
BCR  $AC = 3x - 7$ , and  $BD = 2x + 3$ . Find DM. Use mathematics to explain the process you used to determine your answer. Use words, symbols, or both in your explanation.

DM = 11.5



$$\begin{aligned} 3x - 7 &= 2x + 3 \\ x &= 10 \\ BD &= 2(10) + 3 \\ BD &= 23 \\ DM &= \frac{1}{2}(BD) \\ DM &= \frac{1}{2}(23) \\ DM &= 11.5 \end{aligned}$$

44. Given:  $\overline{AF} \cong \overline{FC}$   
 $\angle ABE \cong \angle EBC$

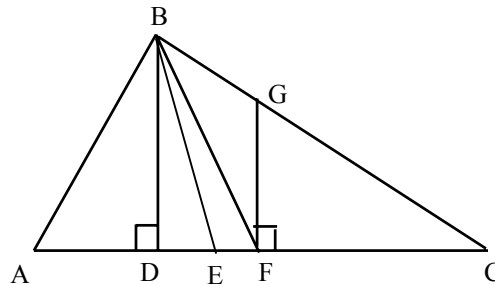
Give the name of each special segment in  $\triangle ABC$ .

$\overline{BD}$ : *Altitude*

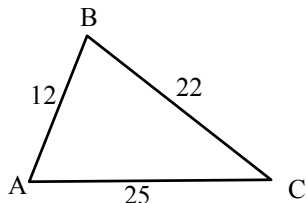
$\overline{BE}$ : *Angle Bisector*

$\overline{BF}$ : *Median*

$\overline{GF}$ : *Perpendicular Bisector*



45. What is the largest angle in the triangle shown? Use mathematics to explain the process you used to determine your answer. Use words, symbols, or both in your explanation.



*$\angle B$  is the largest since it is across from the longest side, 25.*

46. If  $\overline{PT}$  is an altitude of  $\triangle PST$ , what kind of triangle is  $\triangle PST$ ? ***Right triangle***

47. Two sides of a triangle are 6 and 14. What are possible measures of the third side?

$8 < x < 20$

48. What is the sum of the measures of the exterior angles of a heptagon?  $360^\circ$   
 What is the measure of each exterior angle if the heptagon is regular?  $\approx 51.4^\circ$

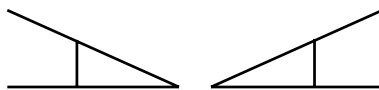
49. If the diagonals of a rhombus are congruent, then it is also what type of quadrilateral?

***Square***

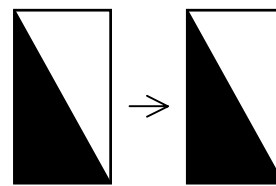
50. Identify each of the following as a **translation**, **reflection**, or **rotation**.



***Rotation***



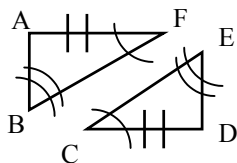
***Reflection***



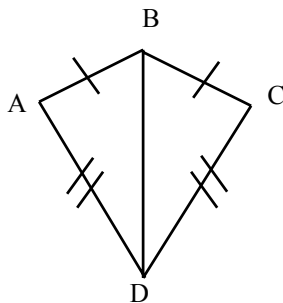
***Translation***

51. If  $\triangle PQR \cong \triangle UVW$ , does it follow that  $\triangle RQP \cong \triangle WVU$ ? *yes*

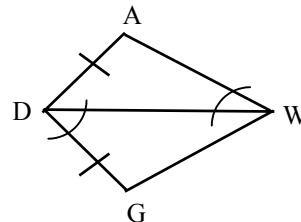
52. For each pair of triangles, determine which triangles are congruent and if so identify the congruence theorem used. If not enough information is available, write cannot be determined.



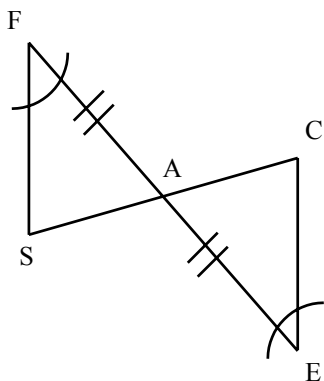
a.  $\triangle BAF \cong \triangle EDC$   
AAS



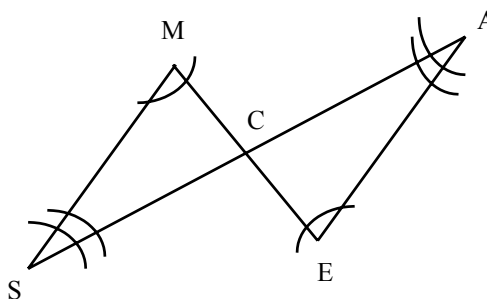
b.  $\triangle BAD \cong \triangle BCD$   
SSS



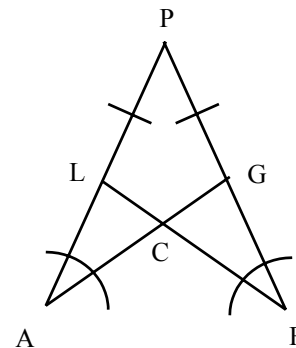
c. Cannot be determined



d.  $\triangle SFA \cong \triangle CEA$   
ASA



e. Cannot be determined

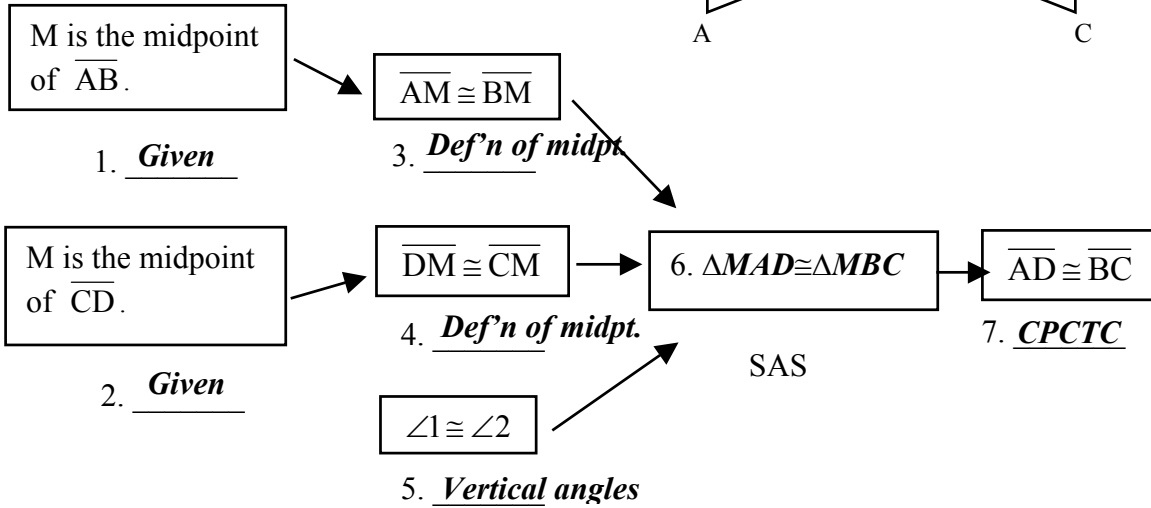
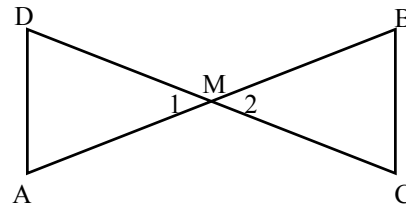


f.  $\triangle PAG \cong \triangle PFL$   
AAS

53. Provide each missing reason or statement in the following flow chart proof.  
BCR

**GIVEN:** M is the midpoint of  $\overline{AB}$ .  
M is the midpoint of  $\overline{CD}$ .

**SHOW:**  $\overline{AD} \cong \overline{BC}$ .

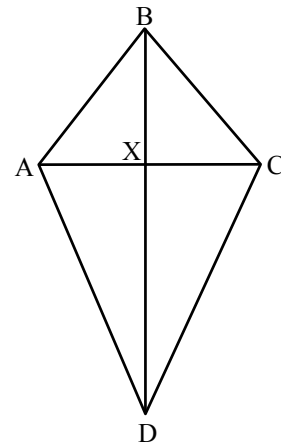


54. Complete the proof given below.

BCR

**GIVEN:** ABCD with  $\overline{AB} \cong \overline{BC}$   
and  $\overline{BD}$  bisects  $\overline{AC}$

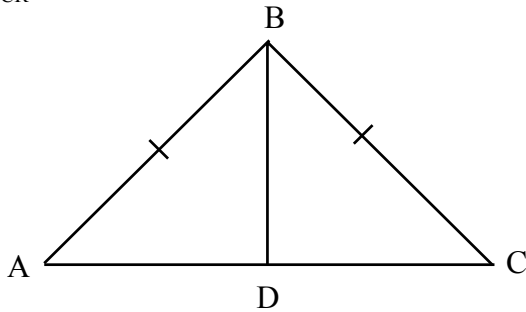
**PROVE:**  $\angle ADB \cong \angle CDB$



| <u>STATEMENTS</u>                          | <u>REASONS</u>             |
|--|----------------------------|
| 1) $\overline{AB} \cong \overline{BC}$     | 1) <b>Given</b>            |
| 2) $\overline{BD}$ bisects $\overline{AC}$ | 2) <b>Given</b>            |
| 3) $\overline{AX} \cong \overline{XC}$     | 3) <b>Def. of Bisector</b> |
| 4) $\overline{BX} \cong \overline{BX}$     | 4) <b>Reflexive</b>        |
| 5) $\triangle ABX \cong \triangle CBX$     | 5) <b>SSS</b>              |
| 6) $\angle ABX \cong \angle CBX$           | 6) <b>CPCTC</b>            |
| 7) $\overline{BD} \cong \overline{BD}$     | 7) <b>Reflexive</b>        |
| 8) $\triangle ADB \cong \triangle CDB$     | 8) <b>SAS</b>              |
| 9) $\angle ADB \cong \angle CDB$           | 9) <b>CPCTC</b>            |

55. Isosceles triangle ABC is shown below.  $\overline{BD}$  is the angle bisector of  $\angle ABC$ .

ECR



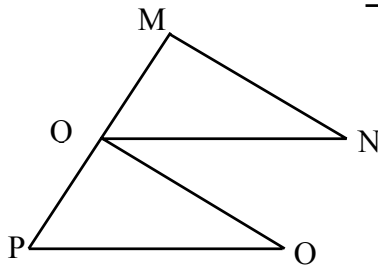
| Statements   | Reasons                   |
|--|---------------------------|
| 1. $\overline{BD}$ is the bisector of $\angle ABC$ . | 1. Given                  |
| 2. $\triangle ABC$ is isosceles                      | 2. Given                  |
| 3. $\overline{AB} \cong \overline{BC}$               | 3. Defn. of isosceles     |
| 4. $\overline{BD} \cong \overline{BD}$               | 4. Reflexive property of  |
| 5. $\triangle ABD \cong \triangle CBD$               | 5. SAS                    |
| 6. $\overline{AD} \cong \overline{CD}$               | 6. CPCTC                  |
| 7. $\overline{BD}$ bisects $\overline{AC}$           | 7. Defn. segment bisector |

Prove that  $\overline{BD}$  bisects  $\overline{AC}$ .

*A sample solution is provided here.  
There are other representations.  
Flow-chart and paragraph proofs  
are also acceptable.*

56. Given: Q is the midpoint of  $\overline{MP}$ ,  $\overline{QN} \parallel \overline{PO}$ , and  $\angle N \cong \angle O$ .

BCR



Prove:  $\angle QMN \cong \angle PQO$ .

| Statements  | Reasons                     |
|---|-----------------------------|
| <b><math>Q</math> is the midpoint of <math>\overline{MP}</math></b> | <b>Given</b>                |
| <b><math>\overline{MQ} \cong \overline{PQ}</math></b>               | <b>Def. of midpoint</b>     |
| <b><math>\overline{QN} \parallel \overline{PO}</math></b>           | <b>Given</b>                |
| <b><math>\angle MQN \cong \angle QPO</math></b>                     | <b>Corresponding angles</b> |
| <b><math>\angle N \cong \angle O</math></b>                         | <b>Given</b>                |
| <b><math>\triangle MQN \cong \triangle PQO</math></b>               | <b>AAS</b>                  |
| <b><math>\angle QMN \cong \angle PQO</math></b>                     | <b>CPCTC</b>                |

*A sample solution is provided here.  
There are other representations.  
Flow-chart and paragraph proofs  
are also acceptable.*

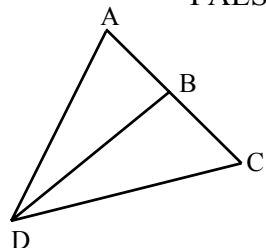
**UNITS I and II:**

Select **always**, **sometimes** or **never** for each statement below. Use mathematics to justify your answer.

- |                                    |                                    |                                    |   |
|------------------------------------|------------------------------------|------------------------------------|---|
| <input checked="" type="radio"/> A | S                                  | N                                  | 57. The medians of an equilateral triangle are also the altitudes.  |
| <input checked="" type="radio"/> A | S                                  | N                                  | 58. If two angles and a non-included side of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent. |
| A                                  | S                                  | <input checked="" type="radio"/> N | 59. The vertices of a triangle are collinear.   |
| <input checked="" type="radio"/> A | S                                  | N                                  | 60. Two intersecting lines are coplanar.  |
| A                                  | <input checked="" type="radio"/> S | N                                  | 61. If an altitude of a triangle is also a median, then the triangle is equilateral.  |
| A                                  | S                                  | <input checked="" type="radio"/> N | 62. A right triangle contains an obtuse angle.  |
| A                                  | S                                  | <input checked="" type="radio"/> N | 63. Two angles of an equilateral triangle are complementary.  |

True or False. Use mathematics to justify your answer.

- F** 64. Making a conjecture from your observations is called deductive reasoning.
- F** 65. The angle bisector in a triangle bisects the opposite side.
- F** 66. A geometric construction uses the following tools: a compass, a protractor, and a straightedge.
- F** 67. ASA and SSA are two shortcuts for showing that two triangles are congruent.
- T** 68. The complement of an acute angle is another acute angle.
- F** 69. Every rhombus is a square.
- F** 70. If the diagonals of a quadrilateral are perpendicular, then the quadrilateral must be a square.
- T** 71. If the base angles of an isosceles triangle each measure  $42^\circ$ , then the measure of the vertex angle is  $96^\circ$ .
- F** 72. If two parallel lines are cut by a transversal, then the corresponding angles formed are supplementary.
- T** 73. If a plane were to intersect a cone, the cross section could be a triangle.
- F** 74. Lengths of 15 cm, 22 cm, and 37 cm could form the sides of a triangle.
- T** 75. The angle bisectors of perpendicular lines are also perpendicular.
76. Given  $\triangle ABD \cong \triangle CBD$ , determine whether the following are TRUE or FALSE.



- |  |                         |   |                         |
|--|-------------------------|---|-------------------------|
| a) $\angle A \cong \angle C$               | <u>    <b>T</b>    </u> | b) $\overline{DB} \perp \overline{AC}$            | <u>    <b>T</b>    </u> |
| c) $\overline{DB}$ bisects $\overline{AC}$ | <u>    <b>T</b>    </u> | d) $\overline{DB}$ bisects $\angle ADC$           | <u>    <b>T</b>    </u> |
| e) $\overline{AD} \cong \overline{DC}$     | <u>    <b>F</b>    </u> | f) $\overline{AD} \cong \overline{DC}$            | <u>    <b>T</b>    </u> |
| g) $\triangle BDA \cong \triangle BDC$     | <u>    <b>F</b>    </u> | h) $\overline{DB}$ is a median of $\triangle ADC$ | <u>    <b>T</b>    </u> |

UNIT III: LOGIC

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77. Define inductive reasoning. *Making a conjecture after looking for a pattern in several examples.*

---

78. Define deductive reasoning. *Using laws of logic to prove statements from known facts.*

---

79. In each example, state the type of reasoning Abdul uses to make his conclusion.

A. Abdul broke out in hives the last four times that he ate chocolate candy. Abdul concludes that he is allergic to chocolate candy. *Inductive*

B. Abdul's doctor's tests conclude that if Abdul eats chocolate, then he will break out in hives. Abdul eats a Snickers bar and therefore Abdul breaks out in hives. *Deductive*

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80. Translate the following expressions into words using the given statements.

P: This month is June.

Q: Summer vacation begins this month.

R: I work during summer vacation.

1)  $\sim P \vee \sim Q$

*This month is NOT June or summer vacation does NOT begin this month.*

2)  $(P \wedge Q) \rightarrow R$

*If this month is June and summer vacation begins this month, then I work during summer vacation.*

3)  $R \leftrightarrow (P \vee Q)$

*I work during summer vacation if and only if this month is June or summer vacation begins this month.*

---

81H. Construct a truth table to determine the truth value of  $\sim (R \vee S) \leftrightarrow (R \wedge \sim S)$ . Indicate whether the statement is a tautology, a contradiction, or neither. **neither**

| $R$ | $S$ | $\sim S$ | $R \vee S$ | $\sim(R \vee S)$ | $R \wedge \sim S$ | $\sim (R \vee S) \leftrightarrow (R \wedge \sim S)$ |
|-----|-----|----------|------------|------------------|-------------------|---|
| $T$ | $T$ | $F$      | $T$        | $F$              | $F$               | $T$   |
| $T$ | $F$ | $T$      | $T$        | $F$              | $T$               | $F$   |
| $F$ | $T$ | $F$      | $T$        | $F$              | $F$               | $T$   |
| $F$ | $F$ | $T$      | $F$        | $T$              | $F$               | $F$   |

82. The statement “If  $\angle 1$  and  $\angle 2$  form a line, then  $m\angle 1 + m\angle 2 = 180^\circ$ ” is true.

- a. State the inverse of the statement in English and give its truth value.  
If  $\angle 1$  and  $\angle 2$  do not form a line, then  $m\angle 1 + m\angle 2 \neq 180^\circ$ . (False)
- b. State the converse of the statement in English and give its truth value.  
If  $m\angle 1 + m\angle 2 = 180^\circ$ , then  $\angle 1$  and  $\angle 2$  form a line. (False)
- c. State the contrapositive of the statement in English and give its truth value.  
If  $m\angle 1 + m\angle 2 \neq 180^\circ$ , then  $\angle 1$  and  $\angle 2$  do not form a line. (True)

83H. If P is true and Q is false, determine the truth value of each of the following.

- a.  $P \wedge \sim Q$      True                      b.  $P \rightarrow Q$      False                      c.  $Q \vee \sim P$      False
- d.  $Q \rightarrow P$      True                                      e.  $P \leftrightarrow Q$      False                      f.  $\sim P \rightarrow Q$      True

84H. Prove with a direct proof.

PREMISE:      $\sim (R \rightarrow \sim W) \rightarrow Y$   
 $\sim S \rightarrow R$   
 $\sim T$   
 $Y \rightarrow T$

CONCLUSION:  $W \rightarrow S$

*A sample solution is provided here.  
 There are other representations.*

| STATEMENTS                                     | REASONS            |
|--|--------------------|
| 1) $\sim (R \rightarrow \sim W) \rightarrow Y$ | 1) <i>Premise</i>  |
| 2) $Y \rightarrow T$                           | 2) <i>Premise</i>  |
| 3) $\sim (R \rightarrow \sim W) \rightarrow T$ | 3) <i>LS (1,2)</i> |
| 4) $\sim T$                                    | 4) <i>Premise</i>  |
| 5) $R \rightarrow \sim W$                      | 5) <i>MT (3,4)</i> |
| 6) $\sim S \rightarrow R$                      | 6) <i>Premise</i>  |
| 7) $\sim S \rightarrow \sim W$                 | 7) <i>LS (5,6)</i> |
| 8) $W \rightarrow S$                           | 8) <i>LC (7)</i>   |



85H. Use a truth table to prove De Morgan's Laws.

| <u>P</u> | <u>Q</u> | <u>~P</u> | <u>~Q</u> | <u>P∧Q</u> | <u>P∨Q</u> | <u>~(P∧Q)</u> | <u>~(P∨Q)</u> | <u>~P∨~Q</u> | <u>~P∧~Q</u> | <u>A</u> | <u>B</u> |
|----------|----------|-----------|-----------|------------|------------|---------------|---------------|--------------|--------------|----------|----------|
| <i>T</i> | <i>T</i> | <i>F</i>  | <i>F</i>  | <i>T</i>   | <i>T</i>   | <i>F</i>      | <i>F</i>      | <i>F</i>     | <i>F</i>     | <i>T</i> | <i>T</i> |
| <i>T</i> | <i>F</i> | <i>F</i>  | <i>T</i>  | <i>F</i>   | <i>T</i>   | <i>T</i>      | <i>F</i>      | <i>T</i>     | <i>F</i>     | <i>T</i> | <i>T</i> |
| <i>F</i> | <i>T</i> | <i>T</i>  | <i>F</i>  | <i>F</i>   | <i>T</i>   | <i>T</i>      | <i>F</i>      | <i>T</i>     | <i>F</i>     | <i>T</i> | <i>T</i> |
| <i>F</i> | <i>F</i> | <i>T</i>  | <i>T</i>  | <i>F</i>   | <i>F</i>   | <i>T</i>      | <i>T</i>      | <i>T</i>     | <i>T</i>     | <i>T</i> | <i>T</i> |

*A:*  $\sim(P \vee Q) \leftrightarrow (\sim P \wedge \sim Q)$

*B:*  $\sim(P \wedge Q) \leftrightarrow (\sim P \vee \sim Q)$

86H. Provide the missing steps and reasons in the proof. The number of steps shown below does not necessarily determine the number of steps needed for the proof. Instead, the outline is only a guide to help you get started.

| <u>PREMISE:</u>    | <u>P</u> → <u>R</u>  | <u>STATEMENTS</u>       | <u>REASONS</u>                            |
|--------------------|----------------------|-------------------------|---|
|                    | <u>T</u> → <u>S</u>  | 1. <u>~R</u>            | 1. <b>Assume the conclusion is false.</b> |
|                    | <u>~T</u> → <u>P</u> | 2. <u>P</u> → <u>R</u>  | 2. <b>Premise</b>                         |
|                    | <u>~S</u>            | 3. <u>~P</u>            | 3. <b>MT (1,2)</b>                        |
| <u>CONCLUSION:</u> | <u>R</u>             | 4. <u>~T</u> → <u>P</u> | 4. <b>Premise</b>                         |
|                    |                      | 5. <u>T</u>             | 5. <b>MT (3,4)</b>                        |
|                    |                      | 6. <u>T</u> → <u>S</u>  | 6. <b>Premise</b>                         |
|                    |                      | 7. <u>S</u>             | 7. <b>MP (5,6)</b>                        |
|                    |                      | 8. <u>~S</u>            | 8. <b>Premise</b>                         |
|                    |                      | 9.                      | 9.  |
|                    |                      | 10.                     | 10.                                       |

*A sample solution is provided here.  
There are other representations.*

But lines 7 and 8 are contradictions and therefore the assumption ~R is false and R is true.