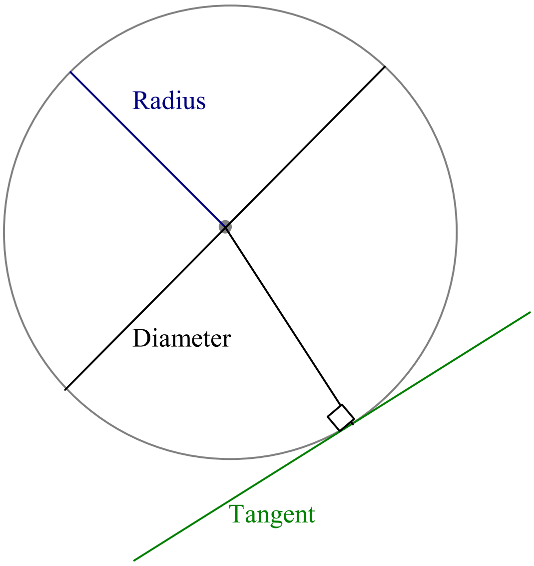
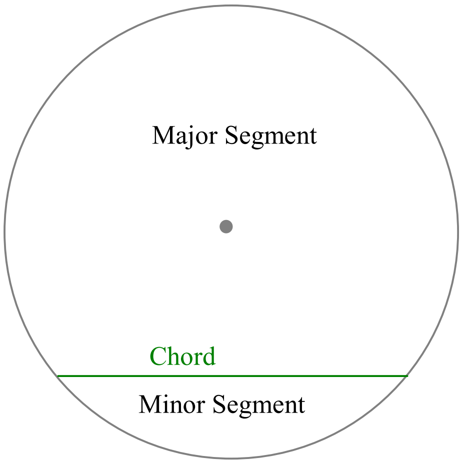
Circle Geometry

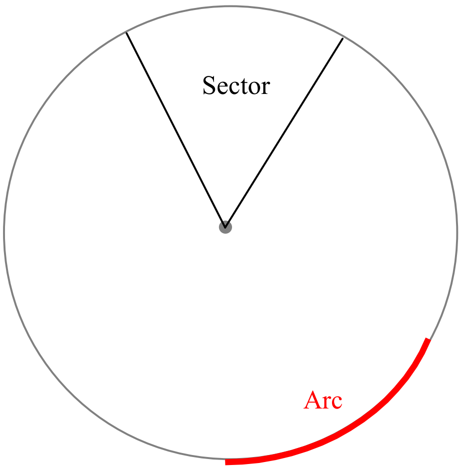
Properties of a Circle
Circle Theorems:

- Angles and chords
- Angles
- Chords
- Tangents
- Cyclic Quadrilaterals

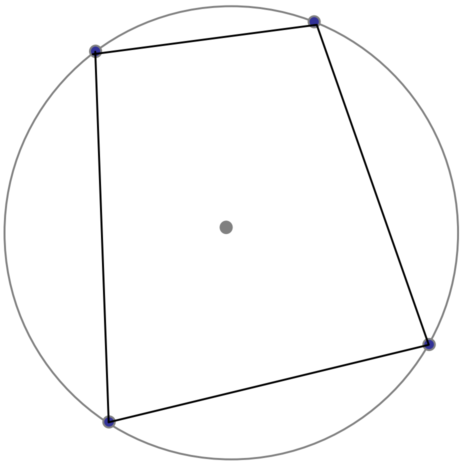
Properties of a Circle

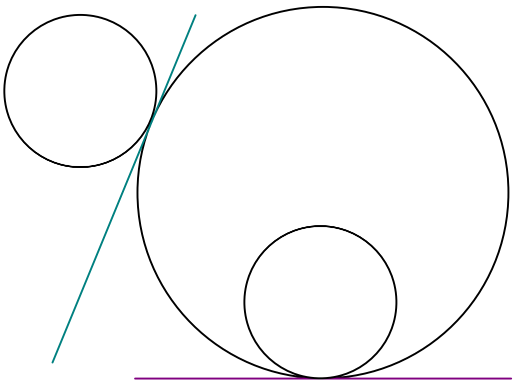




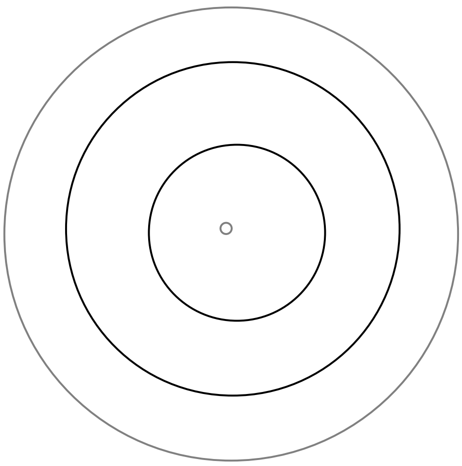


Concyclic points form a Cyclic Quadrilateral



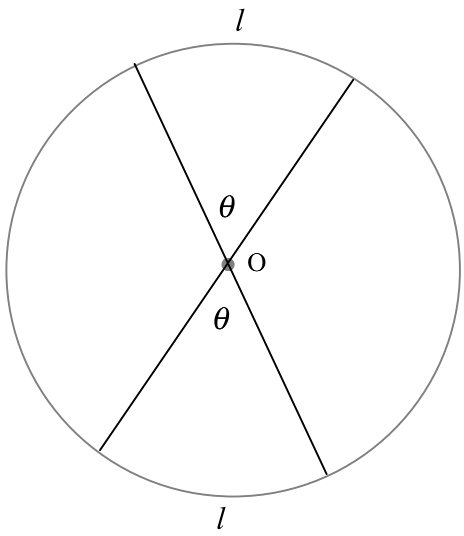
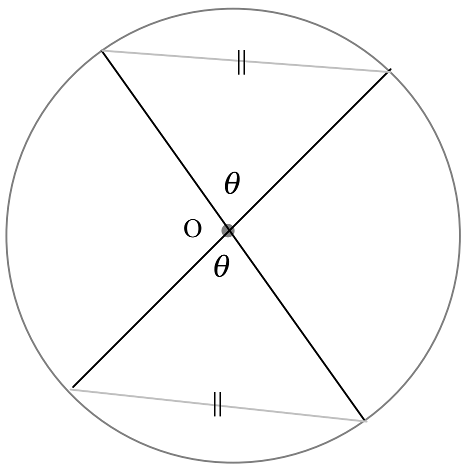
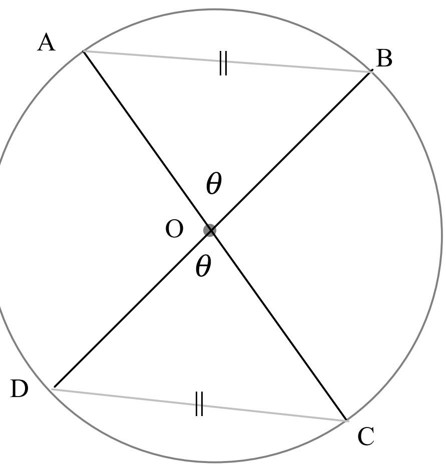


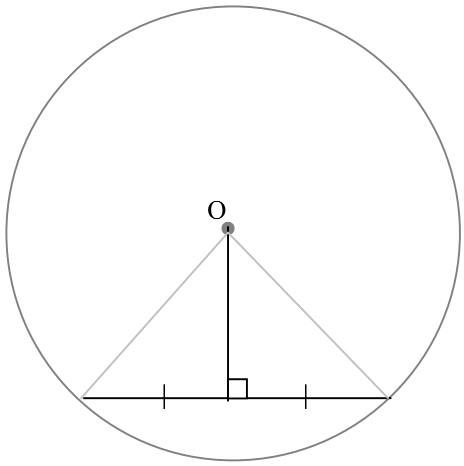
Tangents Externally and Internally



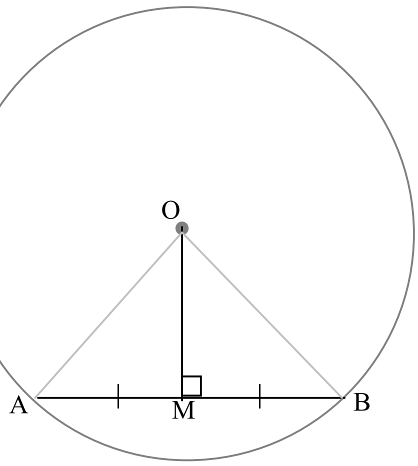
Concentric circles

Circle Theorems

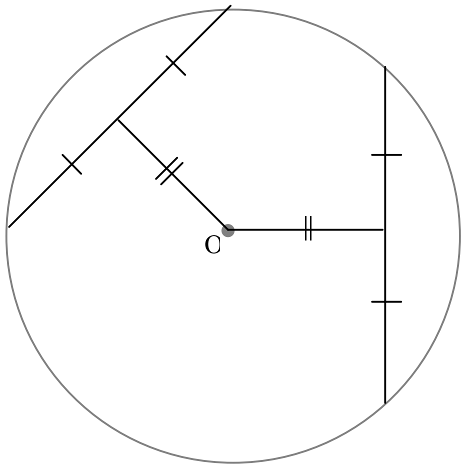
	➢ Equal arcs subtend equal angels at the centre of the circle. ➢ If two arcs subtend equal angles at the centre of the circle, then the arcs are equal. $l = r\theta$
	➢ Equal chords subtend equal angles at the centre of the circle. ➢ Equal angles subtended at the centre of the circle cut off equal chords.
	S $OB = OC$ (radius of circle) A $\angle BOA = \angle COD$ (vert. opp. Angles) S $OA = OD$ (radius of circle) $\therefore \Delta BOA \equiv \Delta COD$ (SAS) AB = DC (corresponding sides in $\equiv \Delta's$)



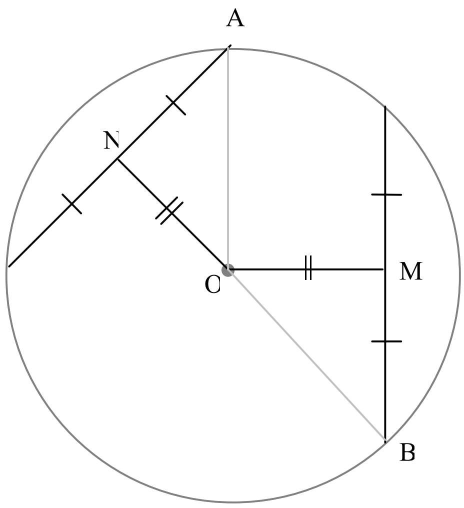
- A perpendicular line from the centre of a circle to a chord bisects the chord.
- A line from the centre of a circle that bisects a chord is perpendicular to the chord.



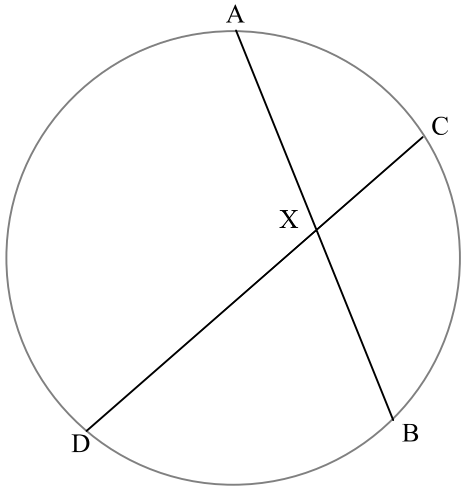
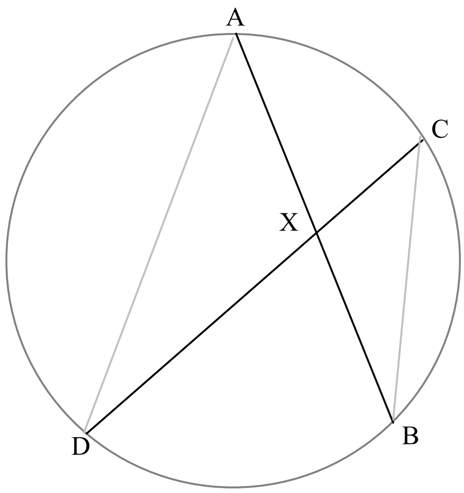
R $\angle OMB = \angle OMA$ (straight line)
H $OB = OA$ (radius of circle)
S $OM = MO$ (common)
$\therefore \Delta AOM \equiv \Delta BOM$ (RHS)
AM = BM (corresponding sides in $\equiv \Delta's$)

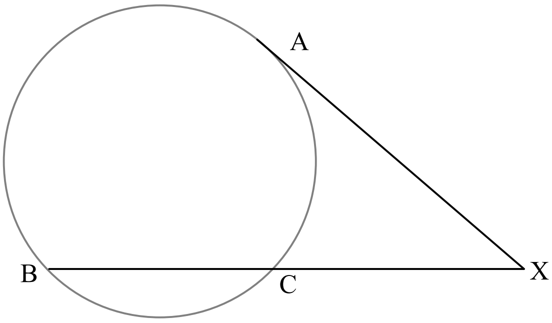


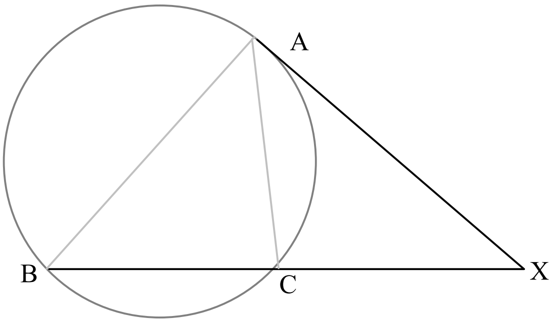
- Equal chords are equidistant from the centre of the circle.
- Chords that are equidistant from the centre are equal.



R $\angle ANO = \angle BMO = 90°$ (A line from the centre of a circle that bisects a chord is perpendicular to the chord)
H $AO = BO$ (Radius of Circle)
S $NO = MO$ (given)
$\therefore \Delta ANO \equiv \Delta BMO$ (RHS)

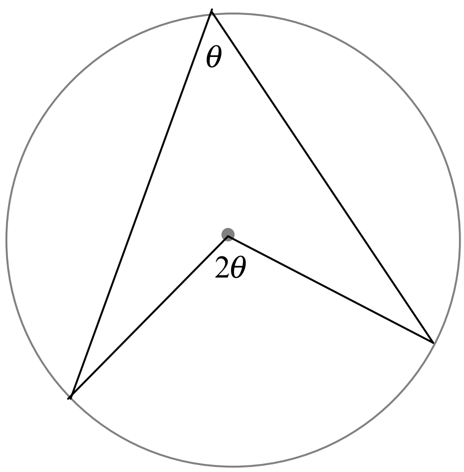
Internally  	➢ The products of intercepts of intersecting chords are equal AX.XB = CX.XD
 	Prove $\Delta AXD \,|||\, \Delta CXD$ A $\angle AXD = \angle CXB$ (vertically opp) A $\angle XAD = \angle XCB$ (Angle standing on the same arc) A $\angle XDA = \angle XBC$ (Angle sum of triangle) Correspond sides $\frac{AX}{XD} = \frac{CX}{XB}$ $\therefore AX.XB = CX.XD$

<table>
<tr>
<td>Externally
</td>
<td>➢ The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point.

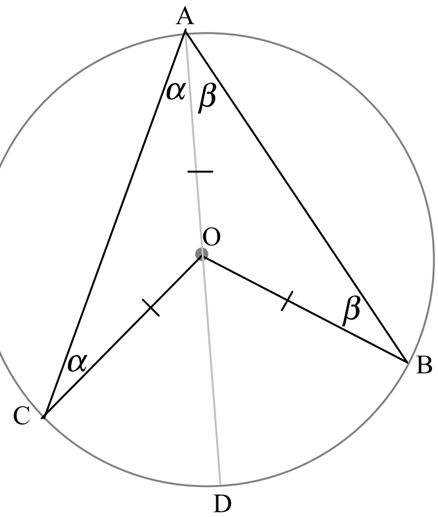
$(AX)^2 = BX.CX$</td>
</tr>
<tr>
<td>Externally
</td>
<td>Prove $\Delta ACX \parallel\!| \Delta BAX$

A $\angle AXC = \angle BXA$ (common)
A $\angle XAC = \angle XBA$ (Angle in alternate segment)
A $\angle ACX = \angle BAX$ (Angle sum of triangle)

Correspond sides
$\frac{AX}{CX} = \frac{BX}{AX}$
$\therefore (AX)^2 = BX.CX$</td>
</tr>
</table>



➢ The angle at the centre of a circle is twice the angle at the circumference subtended by the same arc.



Let $\angle CAO = \alpha$

Let $\angle BAO = \beta$

AO = CO = BO (radius of circle)

$\therefore \angle ACO = \alpha$ (base angles of isosceles Δ)

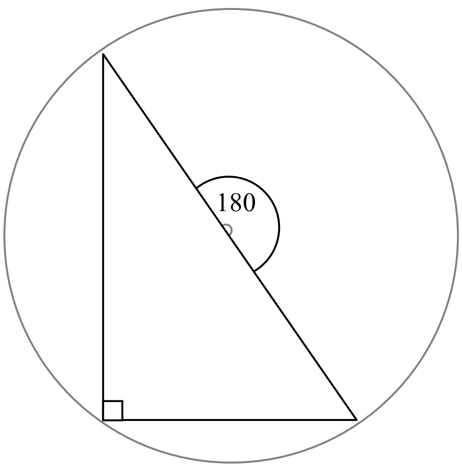
$\therefore \angle ABO = \beta$ (base angles of isosceles Δ)

$\angle COD = 2\alpha$ (exterior angle = two opposite interior angles)

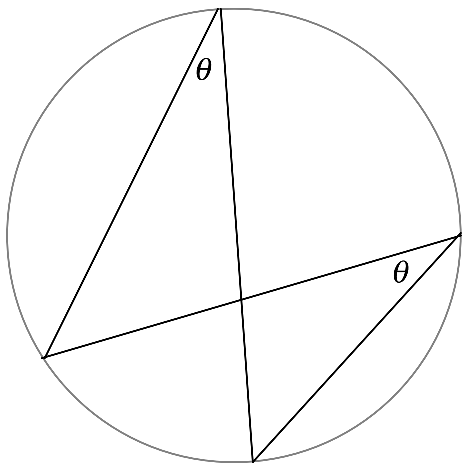
$\angle BOD = 2\beta$ (exterior angle = two opposite interior angles)

$\angle CAB = \alpha + \beta$

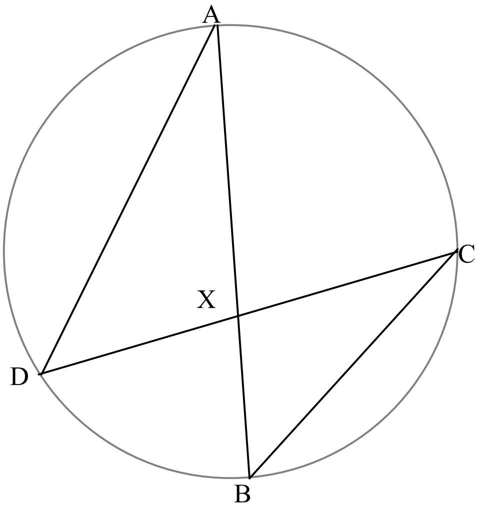
$\angle COD = 2(a + \beta)$



➢ Angle in a semicircle is a right angle.



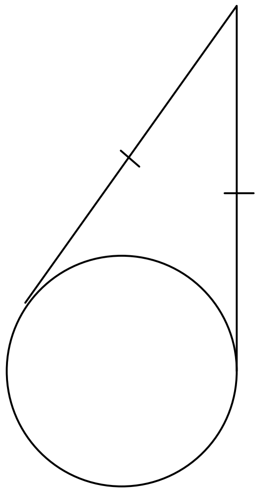
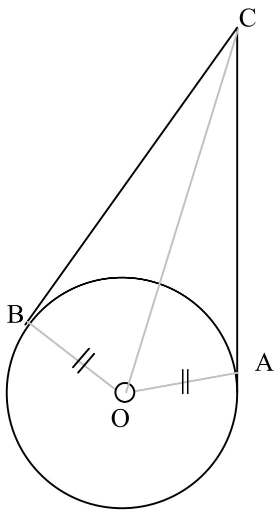
- Angles standing on the same arc are equal.

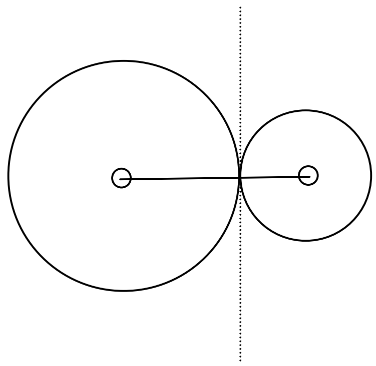
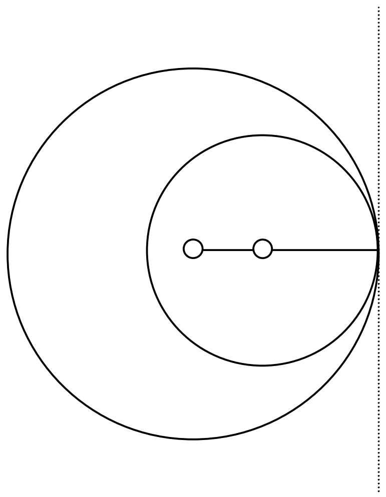


Prove $\Delta AXD ||| \Delta CXD$

A $\angle AXD = \angle CXB$ (vertically opp)
A $\angle XAD = \angle XCB$ (Angle standing on the same arc)
A $\angle XDA = \angle XBC$ (Angle sum of triangle)

Corresponding angles of similar triangles are equal

	➢ Tangents to a circle from an exterior point are equal.
 	Prove $\Delta OAC \equiv \Delta OBC$ R $\angle OAC = \angle OBC$ (90°) H OC = OC (common) S OA =)B (radii) $\therefore$ $\Delta OAC \equiv \Delta OBC$ RHS $\therefore$ AC = BC (corresponding sides in congruent triangles)

	➢ When two circles touch, the line through their centres passes through their point of contact.
	

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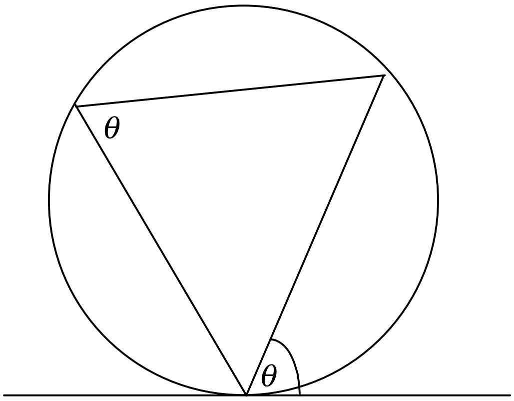
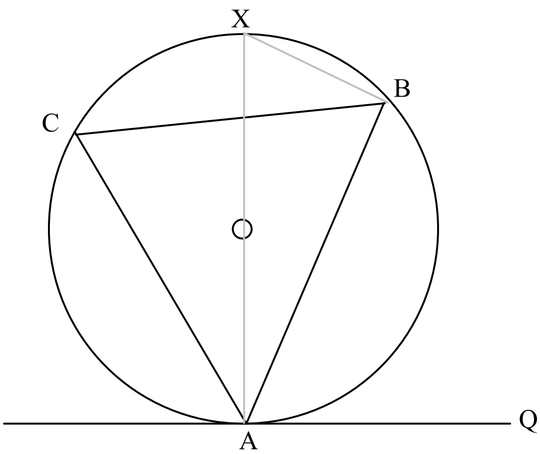
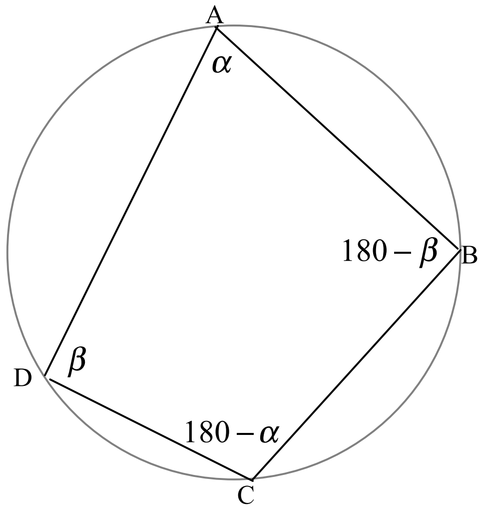
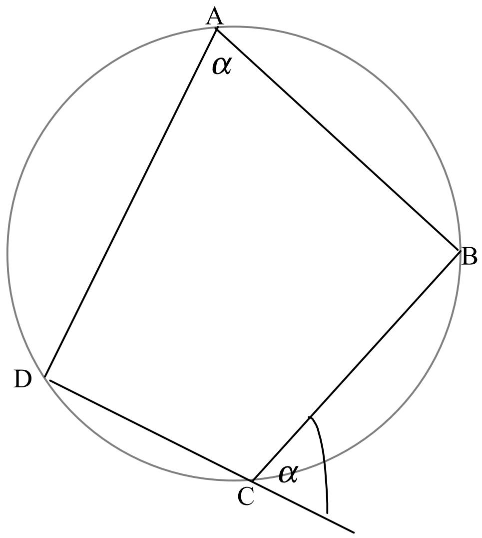
<tr><td></td><td>➢ The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.</td></tr>
<tr><td></td><td>Let $\angle XAB = \alpha$
Let $\angle BAQ = \beta$
$\therefore \alpha + \beta = 90^\circ$

$\angle AXB = \beta$ (angle in semicircle is 90°, complementary angle)
$\angle ACB = \angle AXB$ (angle on the same arc)

$\therefore \angle ACB = \angle BAQ$</td></tr>
</table>

	➢ The opposite angles in a cyclic quadrilateral are supplementary. ➢ If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
	➢ The exterior angle of s cyclic quadrilateral is equal to the interior opposite angle.