

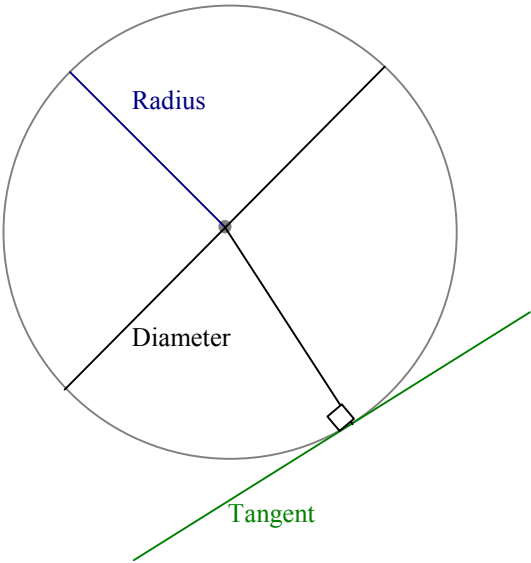
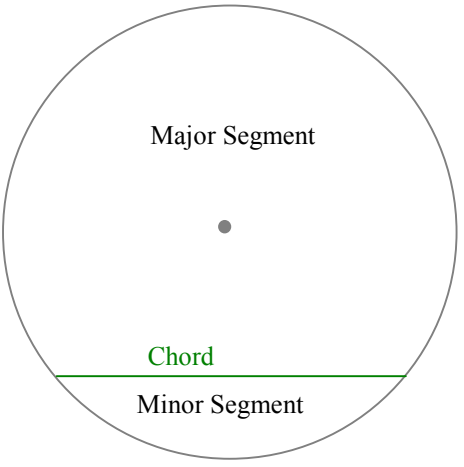
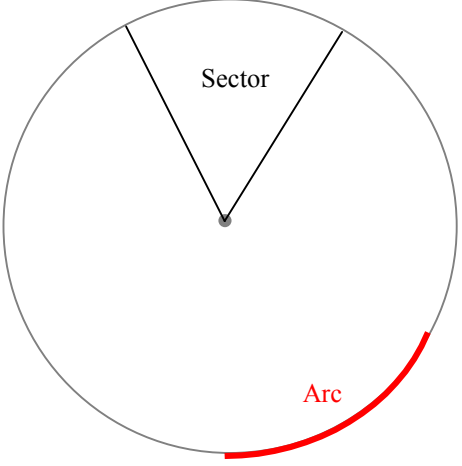
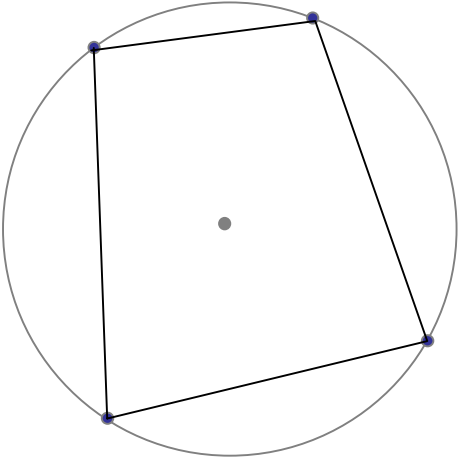
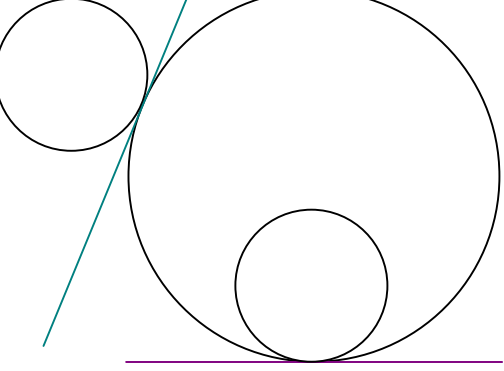
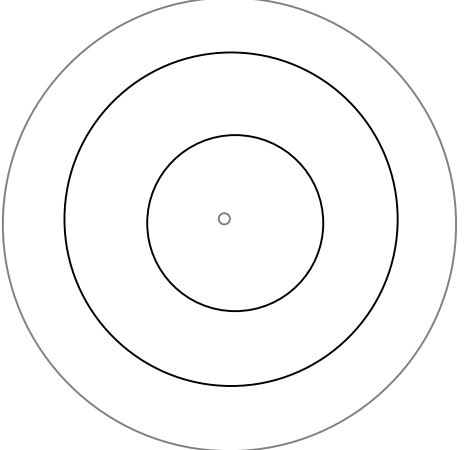
Circle Geometry

Properties of a Circle

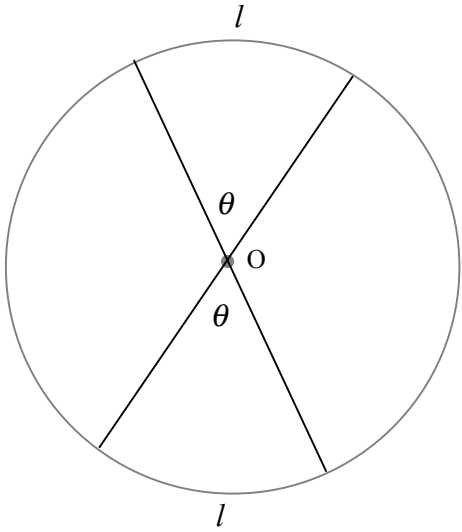
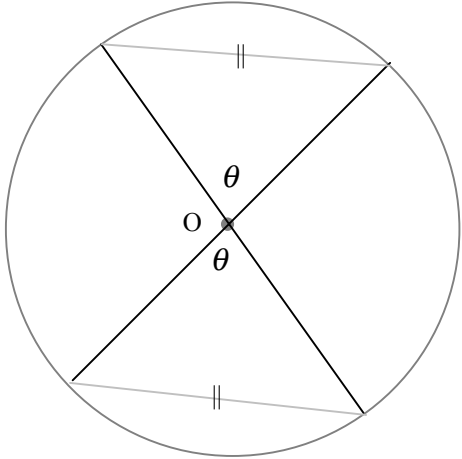
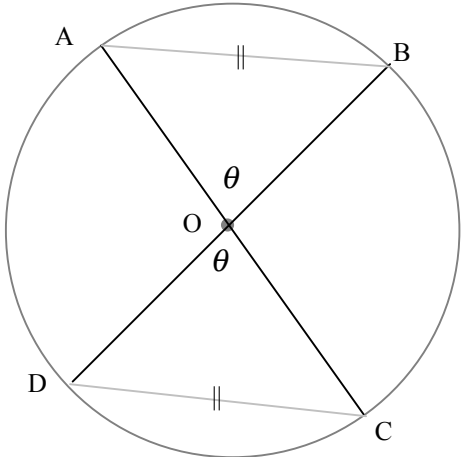
Circle Theorems:

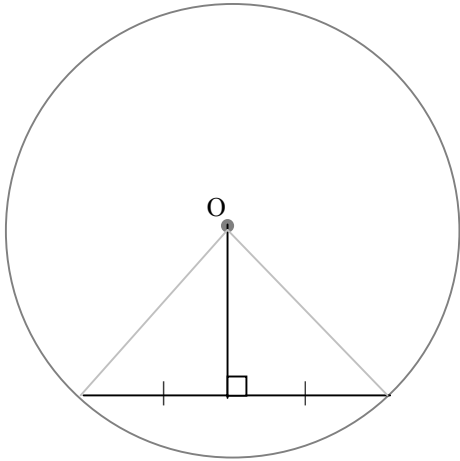
- Angles and chords
- Angles
- Chords
- Tangents
- Cyclic Quadrilaterals

Properties of a Circle

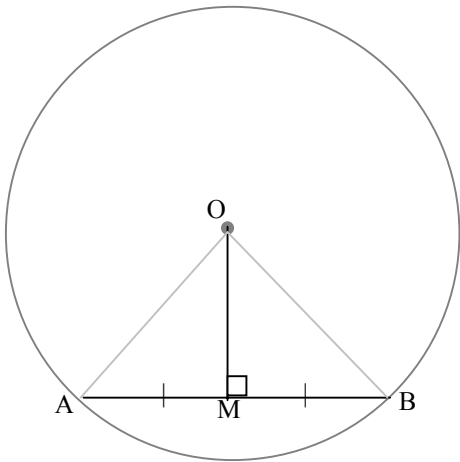
 <p>A circle with a center point. A blue line segment from the center to the circumference is labeled "Radius". A black line segment passing through the center to the circumference is labeled "Diameter". A green line is tangent to the circle at a point, with a right-angle symbol indicating it is perpendicular to the radius at that point. The word "Tangent" is written in green below the line.</p>	 <p>A circle with a center point. A green horizontal line segment connecting two points on the circumference is labeled "Chord". The larger region above the chord is labeled "Major Segment", and the smaller region below the chord is labeled "Minor Segment".</p>
 <p>A circle with a center point. Two radii and the arc between them form a "Sector". A red curved line segment on the circumference is labeled "Arc".</p>	<p>Concyclic points form a Cyclic Quadrilateral</p>  <p>A circle with a center point. Four points on the circumference are connected by straight lines to form a quadrilateral inscribed in the circle.</p>
 <p>Two circles of different sizes. A blue line is tangent to both circles from the outside. A purple horizontal line is tangent to both circles from the inside, touching the bottom of each.</p> <p>Tangents Externally and Internally</p>	 <p>Three concentric circles sharing a common center point.</p> <p>Concentric circles</p>

Circle Theorems

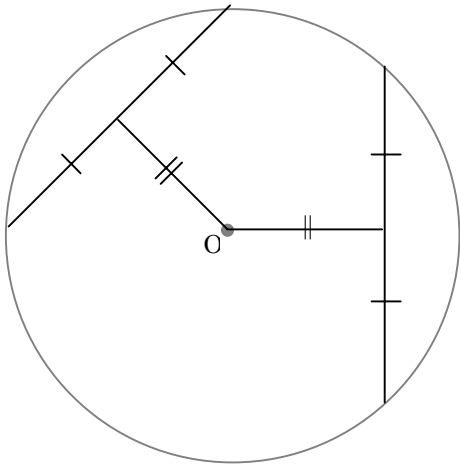
	<ul style="list-style-type: none"> ➤ Equal arcs subtend equal angles at the centre of the circle. ➤ If two arcs subtend equal angles at the centre of the circle, then the arcs are equal. <p>$l = r\theta$</p>
	<ul style="list-style-type: none"> ➤ Equal chords subtend equal angles at the centre of the circle. ➤ Equal angles subtended at the centre of the circle cut off equal chords.
	<p>S $OB = OC$ (radius of circle) A $\angle BOA = \angle COD$ (vert. opp. Angles) S $OA = OD$ (radius of circle) $\therefore \triangle BOA \equiv \triangle COD$ (SAS) $AB = DC$ (corresponding sides in $\equiv \Delta$'s)</p>



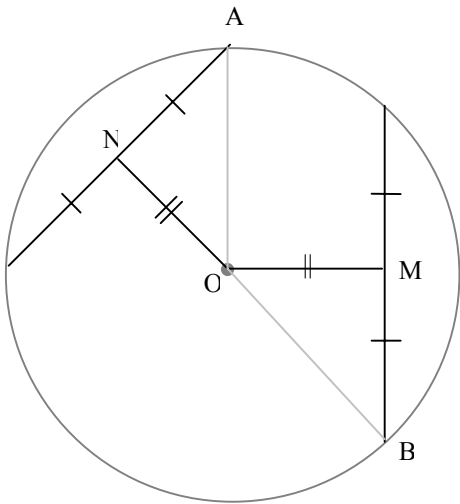
- A perpendicular line from the centre of a circle to a chord bisects the chord.
- A line from the centre of a circle that bisects a chord is perpendicular to the chord.



$\angle OMB = \angle OMA$ (straight line)
 $OB = OA$ (radius of circle)
 $OM = MO$ (common)
 $\therefore \triangle AOM \cong \triangle BOM$ (RHS)
 $AM = BM$ (corresponding sides in $\cong \Delta$'s)

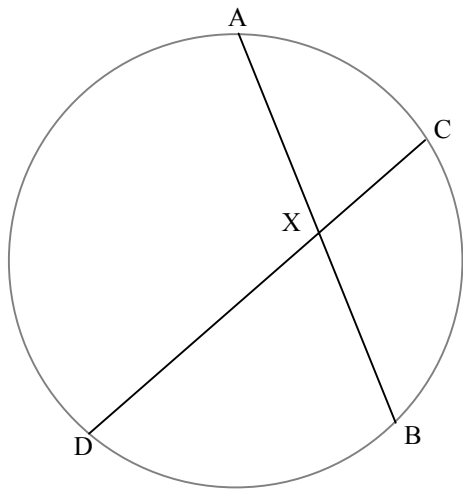


- Equal chords are equidistant from the centre of the circle.
- Chords that are equidistant from the centre are equal.



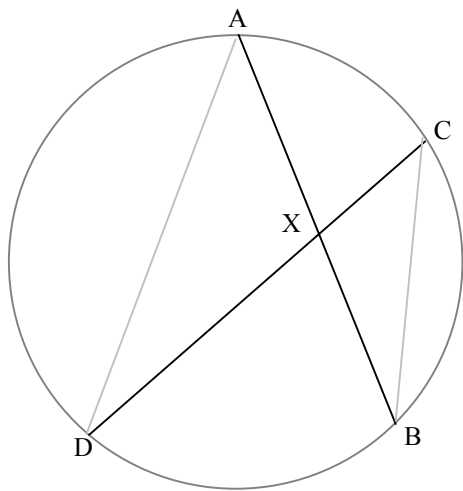
$R \angle ANO = \angle BMO = 90^\circ$ (A line from the centre of a circle that bisects a chord is perpendicular to the chord)
 $H AO = BO$ (Radius of Circle)
 $S NO = MO$ (given)
 $\therefore \triangle ANO \cong \triangle BMO$ (RHS)

Internally



➤ The products of intercepts of intersecting chords are equal

$$AX \cdot XB = CX \cdot XD$$



Prove $\triangle AXD \parallel \triangle CXB$

A $\angle AXD = \angle CXB$ (vertically opp)

A $\angle XAD = \angle XCB$ (Angle standing on the same arc)

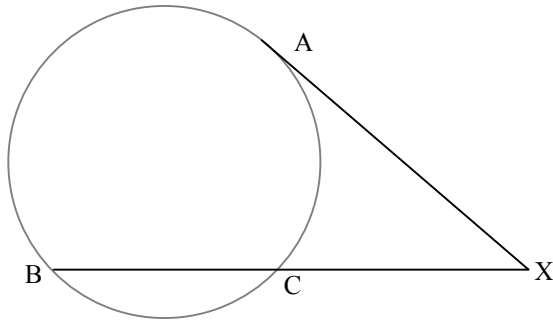
A $\angle XDA = \angle XBC$ (Angle sum of triangle)

Correspond sides

$$\frac{AX}{XD} = \frac{CX}{XB}$$

$$\therefore AX \cdot XB = CX \cdot XD$$

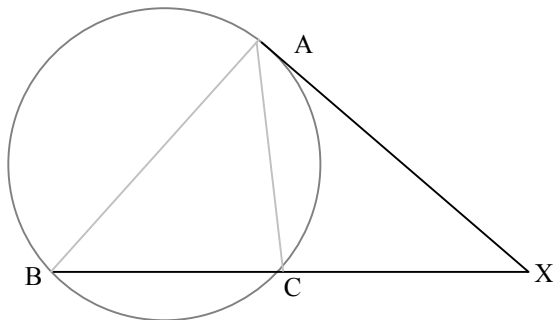
Externally



➤ The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point.

$$(AX)^2 = BX.CX$$

Externally



Prove $\triangle ACX \parallel \triangle BAX$

A $\angle AXC = \angle BXA$ (common)

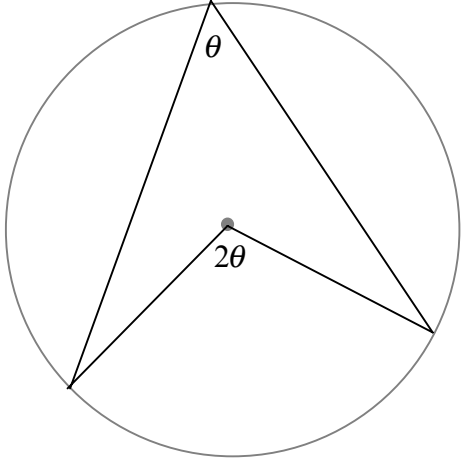
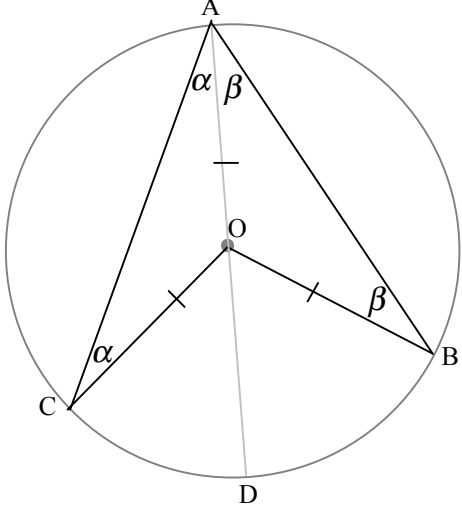
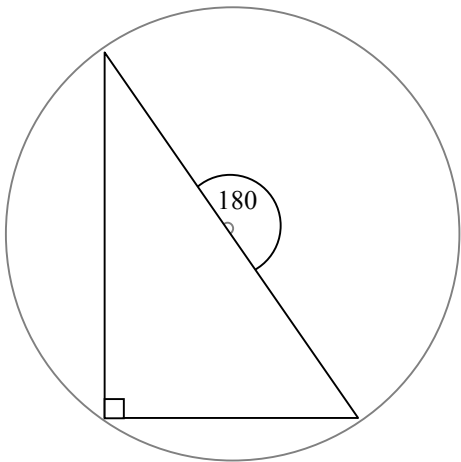
A $\angle XAC = \angle XBA$ (Angle in alternate segment)

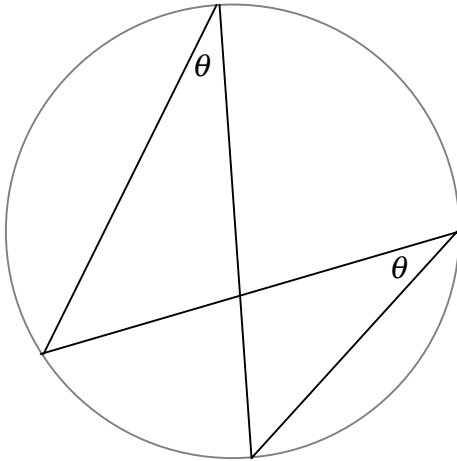
A $\angle ACX = \angle BAX$ (Angle sum of triangle)

Correspond sides

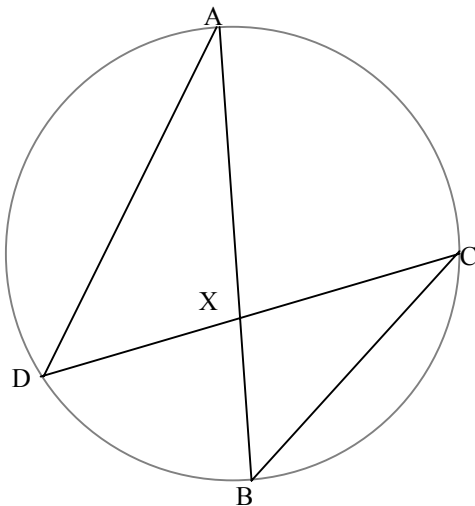
$$\frac{AX}{CX} = \frac{BX}{AX}$$

$$\therefore (AX)^2 = BX.CX$$

	<p>➤ The angle at the centre of a circle is twice the angle at the circumference subtended by the same arc.</p>
	<p>Let $\angle CAO = \alpha$ Let $\angle BAO = \beta$ $AO = CO = BO$ (radius of circle) $\therefore \angle ACO = \alpha$ (base angles of isosceles Δ) $\therefore \angle ABO = \beta$ (base angles of isosceles Δ) $\angle COD = 2\alpha$ (exterior angle = two opposite interior angles) $\angle BOD = 2\beta$ (exterior angle = two opposite interior angles)</p> <p>$\angle CAB = \alpha + \beta$ $\angle COD = 2(\alpha + \beta)$</p>
	<p>➤ Angle in a semicircle is a right angle.</p>



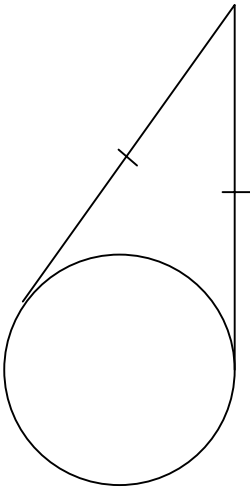
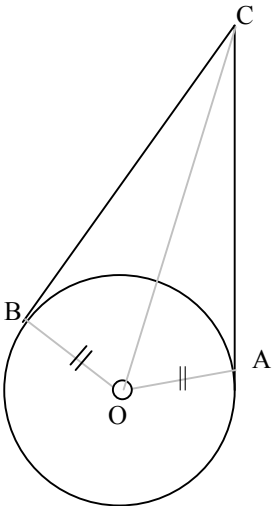
➤ Angles standing on the same arc are equal.

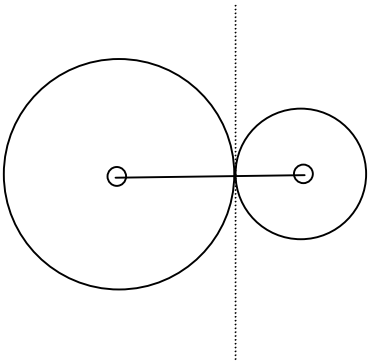
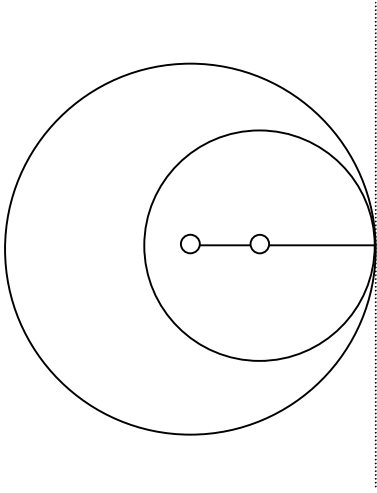


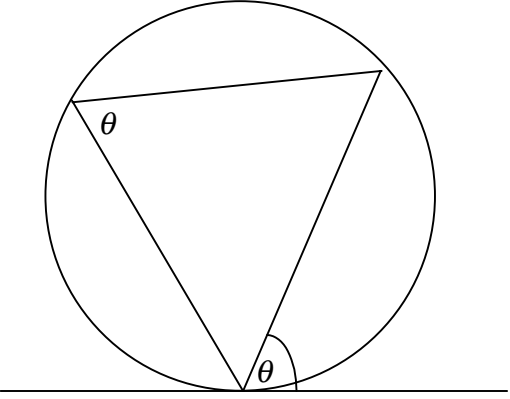
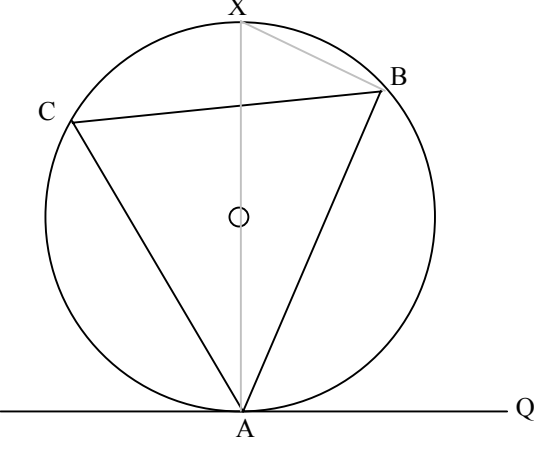
Prove $\triangle AXD \sim \triangle CXB$

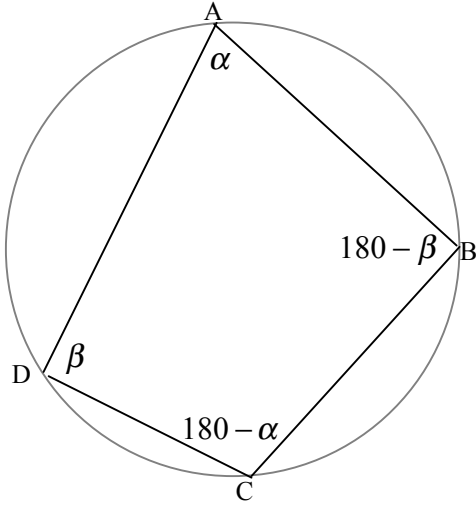
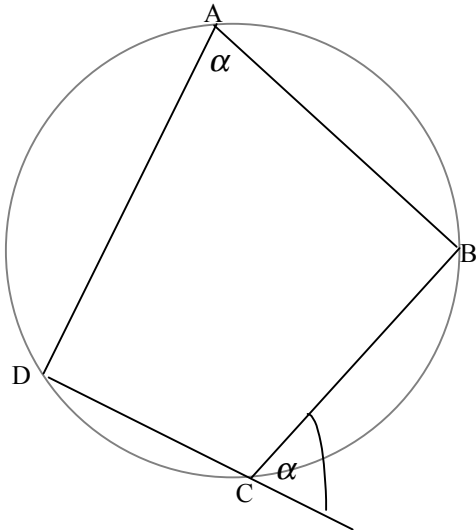
- A $\angle AXD = \angle CXB$ (vertically opp)
- A $\angle XAD = \angle XCB$ (Angle standing on the same arc)
- A $\angle XDA = \angle XBC$ (Angle sum of triangle)

Corresponding angles of similar triangles are equal

	<p>➤ Tangents to a circle from an exterior point are equal.</p>
	<p>Prove $\triangle OAC \equiv \triangle OBC$</p> <p>R $\angle OAC = \angle OBC$ (90°) H $OC = OC$ (common) S $OA = OB$ (radii)</p> <p>$\therefore \triangle OAC \equiv \triangle OBC$ RHS</p> <p>$\therefore AC = BC$ (corresponding sides in congruent triangles)</p>

	<p>➤ When two circles touch, the line through their centres passes through their point of contact.</p>
	

	<p>➤ The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.</p>
	<p>Let $\angle XAB = \alpha$ Let $\angle BAQ = \beta$ $\therefore \alpha + \beta = 90^\circ$</p> <p>$\angle AXB = \beta$ (angle in semicircle is 90°, complementary angle) $\angle ACB = \angle AXB$ (angle on the same arc)</p> <p>$\therefore \angle ACB = \angle BAQ$</p>

	<ul style="list-style-type: none"> ➤ The opposite angles in a cyclic quadrilateral are supplementary. ➤ If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
	<ul style="list-style-type: none"> ➤ The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.